



### Issues in Temporal Planning and Execution

Representation: What kinds of lemporal information can we represent.

 $V = V = (v_1, v_2, \dots, v_n)$ , set of constrained variables  $^{\perp}\mathrm{D}=[\mathrm{D}_{1},\mathrm{D}_{2},\,\,\,_{\mathrm{g}}\,\,,\,\,\mathrm{D}_{\mathrm{b}}]$  , domains for each variable

• <V,D,E>

E = relations on a subset of V: constraints,

representing the legal (partial) solutions

**Constraint Satisfaction** 

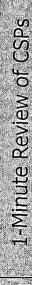
Problems

- Planning
- Generation: How do we construct a temporal plan? Execution
- executed? How do we maintain the stare of the plan given that time is passing (and events are occurring)? - Dispatch. When should the steps in the plan be
- Focus Today: Constraint-Based Models





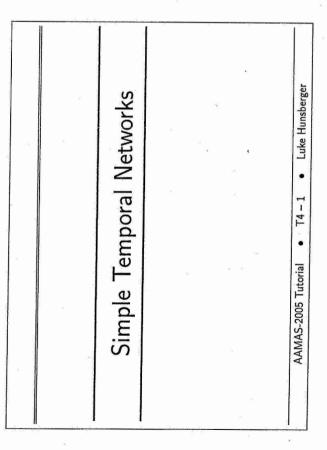


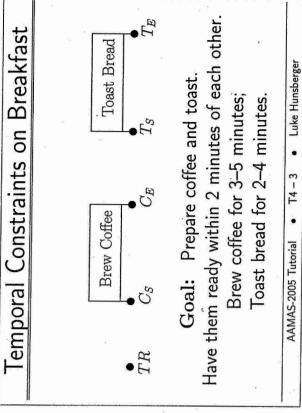


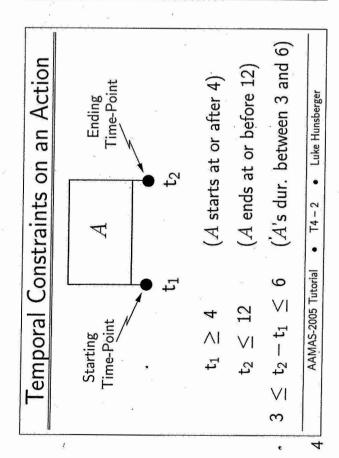


Solve with a combination of search and propagation (forward checking, are consistency, etc.)

•Relations here are binaryhave higher arity as well







# Temporal Constraints on Airline Travel

Goal: Fly from Boston to Seattle:

- Leave Boston after 4 p.m. on Aug. 8;
- Return to Boston before 10 p.m., Aug. 18;
- Away from Boston no more than 7 days;
  - In Seattle at least 5 days; and
- Return flight lasts no more than 7 hours.

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## Simple Temporal Network (STN)\*

A Simple Temporal Network (STN) is a pair,  $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ , where:

- ${\cal T}$  is a set of time-point variables:  $\{t_0,t_1,\ldots,t_{n-1}\}$  and
- $\mathcal C$  is a set of binary constraints, each of the form:  $t_j-t_i\le \delta$ , where  $\delta$  is a real number.

\* (Dechter, Meiri, & Pearl 1991)

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## The Zero Time-Point Variable

- Frequently, it is useful to fix one of the timepoint variables to 0. That "variable" will often be called z.
- Binary constraints involving z are equivalent to unary constraints:

$$t_j - z \le 5 \iff$$

2

VI

$$z-t_1 \le -3 \Leftrightarrow t_1 \ge$$

3

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# Solutions, Consistency, Equivalence

A solution to an STN  $\mathcal{S}=(\mathcal{T},\mathcal{C})$  is a complete set of variable assignments:

$$\left\{t_0=w_0,\;t_1=w_1,\;\ldots,\;t_{n-1}=w_{n-1}\right\}$$

that satisfies all the constraints in  $\mathcal{C}$ .

- An STN with at least one solution is called consistent.
- STNs with identical solution sets are called equivalent.

(A ends before 12)

12

VI

 $t_2 - z$ 

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(A starts after 4)

4

 $z - t_1$ 

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in

# STN for Constrained Action $\mathcal{T} = \{z, t_1, t_2\}, \text{ where: } \begin{aligned} z &= 0 \\ t_1 &= \text{Start of } A \\ t_2 &= \text{ End of } A \end{aligned}$ $\begin{pmatrix} t_2 - t_1 \leq 6 & (\text{Dur. less than 6}) \\ t_1 - t_2 \leq -3 & (\text{Dur. greater than 3}) \end{aligned}$

### STN for Breakfast

$$= \{T_R, C_S, C_E, T_S, T_E\},$$
 where:

$$T_R = 0$$
 (Reference Time-point)  $C_S/C_E = Start/End$  of Coffee Brewing  $T_S/T_E = Start/End$  of Bread Toasting

$$\mathcal{C} = \begin{pmatrix} C_E - C_S \le 5, & C_S - C_E \le -3 \\ T_E - T_S \le 4, & T_S - T_E \le -2 \\ C_E - T_E \le 2, & T_E - C_E \le 2 \\ T_R - C_S \le 0, & T_R - T_S \le 0 \end{pmatrix}$$

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# Graphical Representation of an STN\*

The Distance Graph for an STN,  $\mathcal{S}=(\mathcal{T},\mathcal{C})$ , is a graph,  $\mathcal{G}=(\mathcal{T},\mathcal{E})$ , where:

- Time-points in S correspond to nodes in  $\mathcal{G}$ .
- Constraints in  $\mathcal C$  correspond to edges in  $\mathcal E$ :

$$t_j - t_i \le \delta$$

\* (Dechter, Meiri, & Pearl 1991)

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# Distance Graph for Action Scenario

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## STN for Constrained Air Travel

$$z = \{z, t_1, t_2, t_3, t_4\},$$
 where  $z = \text{Noon, Aug. 8}.$ 

$$|z-t_1| \le -4$$
 (Lv Bos after 4 p.m., 8/8)  $|t_4-z| \le 250$  (Av Bos by 10 p.m., 8/18)

$$t_4 - t_1 \le 168$$
 (Gone no more than 7 days)

$$2-t_3 \le -120$$
 (In Seattle at least 5 days)

$$-t_3 \le 7$$
 (Return flight less than 7 hrs)

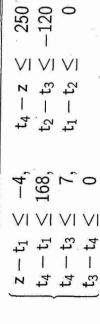
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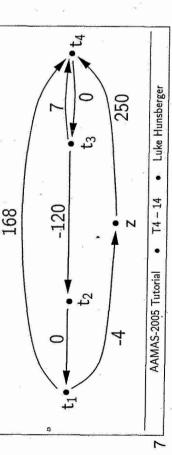
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# Distance Graph for Breakfast $\begin{cases} C_E - C_S \le 5, & C_S - C_E \le -3 \\ T_E - T_S \le 4, & T_S - T_E \le -2 \\ C_E - T_E \le 2, & T_E - C_E \le 2 \\ T_R - C_S \le 0, & T_R - T_S \le 0 \end{cases}$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$ $T_R - C_S \le 0, & T_R - T_S \le 0$

# Explicit constraints in $\mathcal C$ can combine to form implicit constraints: $t_j - t_i \leq 30$ $t_k - t_j \leq 40$ $t_k - t_i \leq 70$ $t_k - t_i \leq 70$ $t_i = ----$

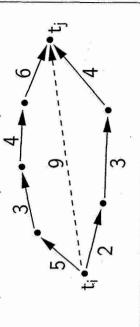
# Distance Graph for Airline Scenario





## mplicit Constraints as Paths

- Chains of implicit constraints in an STN correspond to paths in its Distance Graph.
- Stronger/strongest implicit constraints correspond to shorter/shortest paths.



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### Distance Matrix \*

The Distance Matrix for an STN, S = (T, C), is a matrix D defined by:

Length of Shortest Path  $\mathcal{D}(t_i,t_j) = \text{from } t_i \text{ to } t_j \text{ in the Distance}$  Graph for  $\mathcal{S}$ 

$$\mathcal{D}(t_i,t_j)$$

(Dechter, Meiri, & Pearl 1991)

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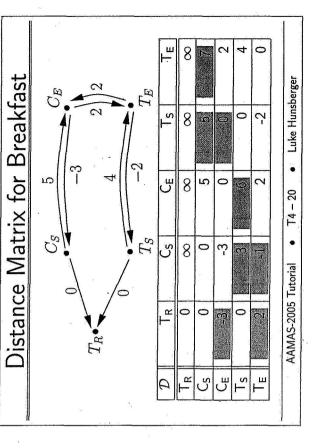
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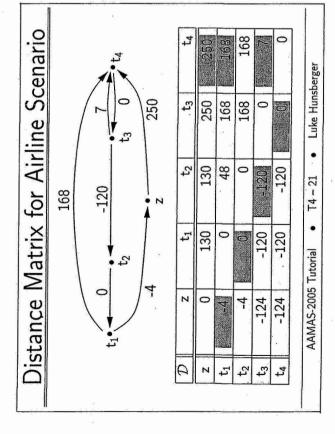
### Distance Matrix (cont'd.)

- The strongest implicit constraint on  $t_i$  and ,  $t_j$  in  ${\cal S}$  is:  ${\sf t_j}-{\sf t_i}\leq {\cal D}({\sf t_i},{\sf t_j})$
- Abuse of notation:  $\mathcal{D}(i,j)$  instead of  $\mathcal{D}(t_i,t_j)$
- D is the All-Pairs, Shortest-Path Matrix for the Distance Graph (Cormen, Leiserson, & Rivest 1990).

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## Computing $\mathcal D$ from Scratch

Polynomial algorithms for computing the All-Pairs, Shortest-Path Matrix (Cormen, Leiserson, & Rivest 1990):

- Floyd-Warshall Algorithm:  $\mathcal{O}(n^3)$
- Johnson's Algorithm:  $\mathcal{O}(n^2 \log n + nm)$

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## Checking Consistency of an STN

Given an STN  $\mathcal S$  with Distance Graph  $\mathcal G$  and Distance Matrix  $\mathcal D$ , the following are equivalent (Dechter, Meiri,  $\mathcal R$  Pearl 1991):

- $\mathcal S$  is consistent.
- Each loop in  $\mathcal G$  has path length  $\geq 0$ .
- The main diagonal of  ${\cal D}$  contains only 0s.

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# Adding Constraint to Consistent STN

- Given: S = (T, C), a consistent STN.
- Adding the new constraint,  $t_j t_i \le \delta$ , to S will maintain the consistency of S iff:

$$-\mathcal{D}(j,i) \leq \delta \quad (i.e., \ 0 \leq \mathcal{D}(j,i) + \delta).$$

Note: This result is stated in different forms by many authors (Dechter, Meiri, & Péarl 1991; Demetrescu & Italiano 2002; Tsamardinos & Pollack 2003; Hunsberger 2003; Rohnert 1985).

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## Rigidly Connected Time-Points

For consistent STNs, the following are equivalent:

- $(t_j t_i) = \delta$ , for some  $\delta$ .
- $\mathcal{D}(i,j) + \mathcal{D}(j,i) = 0$
- ti and tj belong to a loop of path-length 0.

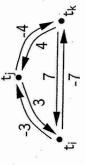


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# Rigidly Connected Time-Points (ctd.)

- $t_i$  and  $t_j$  are said to be *rigidly connected* if  $\mathcal{D}(i,j) = -\mathcal{D}(j,i)$ .
- A set of time-points that are pairwise rigidly connected form a *rigid component*.



Note: Many authors consider rigidly connected time-points and rigid components (Tsamardinos, Muscettola, & Morris 1998; Gerevini, Perini, & Ricci 1996; Wetprasit & Sattar 1998).

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# Adding Constraints to Consistent STNs Result of adding the constraint, $t_j - t_i \le \delta$ : Consistent, Rigid Consistent, Redundant Inconsistent Redundant Consistent C

## Finding a Solution to an STN\*

While some time-points in are *not* rigid with z, Pick some  $t_i$  not rigidly connected to z. Pick some  $\delta \in [-\mathcal{D}(t_i,z), \, \mathcal{D}(z,t_i)]$ . Add the constraint,  $t_i = \delta$ 

\* This algorithm derives from Dechter et al. (1991).

(i.e.,  $t_i - z \le \delta$  and  $z - t_i \le -\delta$ ).

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# Collapsing Rigid Components: Example to the first state of the Hunsberger to the Hu

## Collapsing Rigid Components

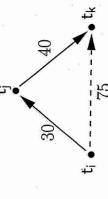
- Select one time-point from each rigid component to serve as its representative
- Re-orient edges involving non-representative members of rigid components
- Associate additional information with each representative sufficient to enable reconstruction of its rigid component

(Tsamardinos, Muscettola, & Morris 1998; Gerevini, Perini, & Ricci 1996; Wetprasit & Sattar 1998).

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### Dominated Constraints

An explicit constraint, c:  $t_j - t_i \le \delta$ , in an STN S is said to be *dominated* in S if *removing* c from S would result in no change to the distance matrix D.



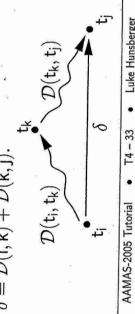
Note: Tsamardinos (1998) defines a different notion of dominance.

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## Dominated Constraints (cont'd.)

If  $\mathcal S$  is consistent and has no rigid components then:

- If  $\mathcal{D}(i,j) < \delta$ , then c is dominated in  $\mathcal{S}$ .
- If  $\mathcal{D}(i,j)=\delta$ , then c is dominated in  $\mathcal{S}$  iff there is some time-point  $t_k \in \mathcal{T}$  such that:  $\delta = \mathcal{D}(i,k) + \mathcal{D}(k,j)$



#### Remove all dominated edges from the (unique) Convert rigid components to cyclical form. Canonical Form of an STN non-rigid remainder of the STN. RIGID COMPONENTS



\* (Hunsberger 2002b)

### Undominated Constraints

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74 - 33

If  ${\mathcal S}$  has no rigid components, then the set of undominated constraints in  ${\mathcal S}$  is uniquely defined and represents the fewest constraints in (Hunsberger 2002b) any STN equivalent to S.

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# Incremental Algs for Distance Matrix

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# Computing Dist. Matrix Incrementally

- Incremental algorithms compute changes resulting from adding a single constraint.
- A naïve incremental algorithm can compute such changes in  $\mathcal{O}(n^2)$  time.
- Better incremental algorithms based on constraint propagation—still  $\mathcal{O}(n^2)$ .

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# Naïve Incremental Algorithm For each entry, $\mathcal{D}(r,s)$ , If $\mathcal{D}(r,i) + \delta + \mathcal{D}(j,s) < \mathcal{D}(r,s)$ , then set $\mathcal{D}(r,s) = \mathcal{D}(r,i) + \delta + \mathcal{D}(j,s).$ $t_r$ $\mathcal{D}(r,s) = \mathcal{D}(r,s)$ $\mathcal{D}(r,s)$ $t_r$ $\mathcal{D}(r,s)$ $\delta$ $\lambda_{AMAS-2005 \ Tutorial} \qquad T4-39 \qquad Luke \ Hunsberger$

# Adding a Constraint to Consistent STN

Given: New constraint c:  $t_j - t_i \le \delta$ .

- Case 1:  $\delta < -\mathcal{D}(j,i)$ . Inconsistent!
  - Case 2:  $\delta \geq \mathcal{D}(i,j)$ . Redundant!
    - Case 3:  $\delta \in [-\mathcal{D}(j,i), \mathcal{D}(i,j))$ .
- Adding c would require updating  $\mathcal{D}$ .
- ⇒ Incremental algorithms focus on Case 3.

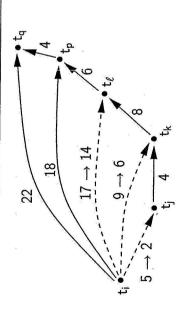
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# Constraint Propagation Algorithm\*

- Propagate updates to  ${\cal D}$  along edges in graph.
- Only propagate along *tight* edges. (Note:  $t_s t_r \le \delta$  is tight iff  $\mathcal{D}(r,s) = \delta$ .)
- Phase I: prop. forward; Phase II: prop. bkwd.
- Checks no more than  $k*\Delta$  cells of  $\mathcal{D}$ , where:  $\Delta =$  number of cells needing updating; and k = max num edges incident on any node.
- $^{*}$  This algorithm is based on the work of several authors (Rohnert 1985; Even & Gazit 1985; Ramalingam & Reps 1996).

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## Propagating Forward



 $\label{eq:continuity} \mbox{Adding the edge, } t_j - t_i \leq 2, \mbox{ requires updating } \\ \mathcal{D}(i,j), \ \mathcal{D}(i,k) \mbox{ and } \mathcal{D}(i,\ell), \mbox{ but not } \mathcal{D}(i,p).$ 

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## Improvements to Incremental Alg.

- Maintain canonical form of STN.
- Only update  $\mathcal D$  for non-rigid portion of STN.
- Propagate only along undominated edges.
- Case 3.1:  $\delta > -\mathcal{D}(\mathsf{j},\mathsf{i})$ . (No new rigidities)
- Case 3.2:  $\delta = -\mathcal{D}(j, i)$ . (New rigidity(ies))

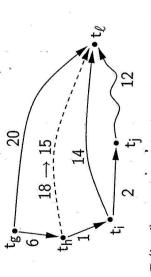
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Case 3.1

The Gory Details –

### Propagating Backward

For each  $t_\ell$  such that  $\mathcal{D}(i,\ell)$  changed during Forward Propagation, propagate backward from  $t_i$ :



Here,  $\mathcal{D}(\mathsf{h},\ell)$  needs updating, but not  $\mathcal{D}(\mathsf{g},\ell)$ .

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 $\label{eq:encounteredTPs} \textit{EncounteredTPs}, \ \, \text{an empty hash-table}.$   $(t_j - t_i \le \delta), \ \, \text{a new constraint where:} \ \, -\mathcal{D}(j,i) < \delta < \mathcal{D}(i,j).$ 

For each  $t_r \in \mathcal{T}$ ,  $Succs(t_r) = \{(t_s - t_r \le \delta_{rs}) \in \mathcal{C}^u\}$  (a hash-table). For each  $t_r \in \mathcal{T}$ ,  $Precs(t_r) = \{(t_r - t_q \le \delta_{qr}) \in \mathcal{C}^u\}$  (a hash-table).

AffectedTPs, an empty hash-table.

 $\mathcal{S} = (\mathcal{T}, \mathcal{C}^{\mathrm{u}})$ , an STN with only *undominated* constraints.

Inputs to Prop<sub>3.1</sub>:

 $\mathcal{D}$ , the distance matrix for  $\mathcal{S}$  (an array).

Note: This algorithm most closely resembles that of Ramalingam and Reps (1996).

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#### Case 3.1 (cont'd.) PropFwd( $t_j$ ), which adds time-points to Affected TPs. Luke Hunsberger Clear Encountered TPs hash-table. T4 - 45The Gory Details – For each t<sub>v</sub> ∈ AffectedTPs, Insert t<sub>j</sub> into Affected TPs. PropBkwd $(t_i, t_v)$ . AAMAS-2005 Tutorial Set: $\mathcal{D}(i,j) = \delta$ . Prop3.1()

 $\operatorname{PropFwd}(t_v)$ , where a path from  $t_i$  to  $t_y$  has already and  $\mathcal{D}(i,y)$  has been updated to the value  $\delta + \mathcal{D}(j,y)$ 

The Gory Details — Case 3.1 (

(cont'd.

For each  $t_z \in Succs(t_y)$ ,

If t<sub>z</sub> ∉ EncounteredTPs,

Insert t<sub>z</sub> into EncounteredTPs If  $\mathcal{D}(j, y) + \delta_{yz} = \mathcal{D}(j, z)$ ,

Remove t<sub>z</sub> from Succs(t<sub>i</sub>) (if in there) If  $\delta + \mathcal{D}(j, y) + \delta_{yz} \le \mathcal{D}(i, z)$ 

Remove t<sub>i</sub> from Precs(t<sub>z</sub>) (if in there) If  $\delta + \mathcal{D}(j, y) + \delta_{yz} < \mathcal{D}(i, z)$ ,

Set:  $\mathcal{D}(i, z) = \delta + \mathcal{D}(j, y) + \delta_{yz}$ 

Insert tz into Affected TPs

PropFwd( $t_z$ ).

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### cont'd. Case 3.1 The Gory Details —

 $\mathbf{PropBkwd}(t_s,t_v),$  where a path from  $t_s$  to  $t_v$  has already been processed and  $\mathcal{D}(s,v)$  has been updated to the value  $\mathcal{D}(s,i)+\delta+\mathcal{D}(j,v)$ .

For each  $t_r \in Precs(t_s)$ ,

If t<sub>r</sub> ∉ EncounteredTPs,

Insert tr into EncounteredTPs If  $\delta_{rs} + \mathcal{D}(s, i) = \mathcal{D}(r, i)$ ,

If  $\delta_{rs} + \mathcal{D}(s, i) + \mathcal{D}(i, v) \leq \mathcal{D}(r, v)$ ,

Remove t<sub>v</sub> from Succs(t<sub>r</sub>) (if in there) Remove t, from Precs(t<sub>v</sub>) (if in there)

Set:  $\mathcal{D}(r, v) = \delta_{rs} + \mathcal{D}(s, i) + \mathcal{D}(i, v)$ If  $\delta_{rs} + \mathcal{D}(s,i) + \mathcal{D}(i,v) < \mathcal{D}(r,v)$ 

PropBkwd(tr, tv)

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## Case 3.2: Creating New Rigidity

Adding constraint,  $t_j - t_i \le -\mathcal{D}(j, i)$ .

- Determine newly rigid time-points.
- Collapse new rigid component down to two points, using t<sub>i</sub> as rep. for incoming edges and t; as rep. for outgoing edges.
- Update set  $C^{u}$  of undominated constraints.
- Run Prop<sub>3.1</sub> algorithm.
- Collapse t; and t; into a single point.

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T4 - 50

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### Further Reading

- Demetrescu and Italiano (2001; 2002) consider special cases where each edge can assume a bounded number of values; or where all edge weights are non-negative.
- Ramalingham and Reps (1996) introduce incremental complexity analysis.
- Zaroliagis (2002) discusses incremental and decremental algorithms.

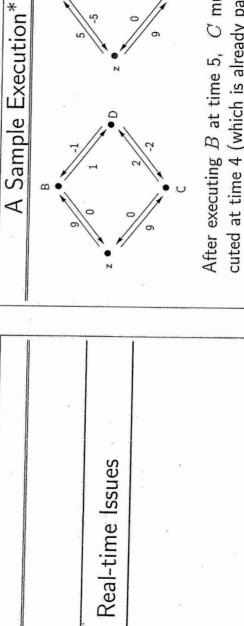
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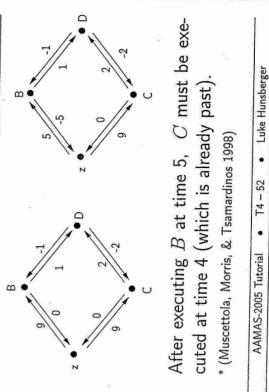
## Executing a Temporal Network

- To execute a time-point means to assign that time-point to the current moment.
- Goal: Maintain consistency of network while executing its time-points.
- Challenges:

Decisions must be made in real time. Updating  $\mathcal D$  takes time.

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### Greedy Dispatcher\*

While some time-points not yet executed:

Wait until some time-point is executable.

If more than one, pick one to execute. Propagate updates only to neighboring time-points (i.e., do no fully update  $\mathcal{D}$ ).

\* (Muscettola, Morris, & Tsamardinos 1998)

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# Lower and Upper Dominance\* A. $\delta < 0$ $\phi < 0$ B. $\sigma < 0$ $\sigma > 0$

- The negative edge AC is lower-dominated if:  $\delta = \phi + \mathcal{D}(B,C)$ .
- The non-negative edge UW is upperdominated if:  $\delta = \mathcal{D}(U,V) + \phi$ .
- \* (Muscettola, Morris, & Tsamardinos 1998)

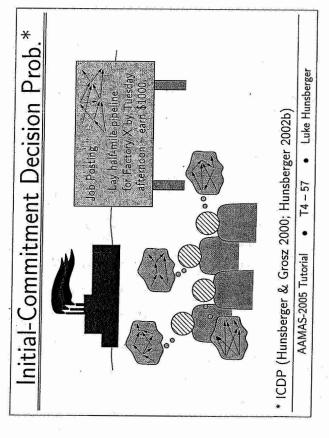
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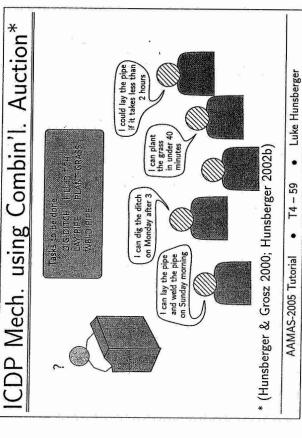
### Dispatchability\*

- An STN that is guaranteed to be satisfied by Greedy Dispatcher is called *dispatchable*.
- Any consistent STN can be transformed into an equivalent dispatchable STN.
- Step I: The corresponding AII-Pairs graph is equivalent and dispatchable.
- Step II: Remove *lower/upper-dominated* edges (does not affect dispatchability).
- \* (Muscettola, Morris, & Tsamardinos 1998)
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# Collaborative Planning with STNs

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### The ICDP - in Words

- A group of agents, each with pre-existing commitments subject to temporal constraints
- A new opportunity for group action (a set of tasks also subject to temporal constraints)
- Agents must reason locally and globally about whether to commit (alone and together) to the proposed action.

## ICDP Mechanism - in Words

- Agents (reasoning locally) bid on subsets of tasks in group activity: a *combinatorial* auction (Rassenti, Smith, & Bulfin 1982).
- Agents include temporal constraints in their bids to protect their pre-existing commitments.
- Global goal: find an awardable set of bids (each task covered by some bid; temporal constraints in bids jointly satisfiable).

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74 - 58

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## Problems to Solve re: ICDP

- Bid Generation:
- Select tasks and generate protective temporal constraints
- Winner Determination:
- Find an awardable set of bids.
- Post-Auction Coordination:

Deal with temporal dependencies among tasks being done by different agents without requiring excessive communication overhead.

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#### 

## Bid Generation using STNs

Proposed Group Activity  $S_Y = (T_X, \mathcal{C}_Y)$   $S_X = (T_X, \mathcal{C}_X)$ 



 $\mathcal{S}_Z = (\mathcal{T}_Z, \mathcal{C}_Z) = (\mathcal{T}_X \cup \mathcal{T}_Y, \ \mathcal{C}_X \cup \mathcal{C}_Y) \ \text{is consistent}$ 

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if  $S_X$  and  $S_Y$  are (since they only share z).

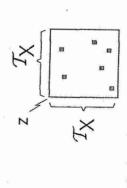
# Bid Generation using STNs (cont'd.)

  $\mathcal{C}_B^x = \{t_j - t_i \leq \mathcal{D}_B(i,j) \mid t_i, t_j \in \mathcal{T}_X\} \text{ would suffice (in bid) to protect agent's pre-existing commitments.}$ 

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# Bid Generation using STNs (cont'd.)

... but necessary to include only edges in canonical form of  $(\mathcal{T}_X, \mathcal{C}_B^x)$  that are stronger than the corresponding edges in  $\mathcal{S}_X = (\mathcal{T}_X, \mathcal{C}_X)$  — i.e., edges for which  $\mathcal{D}_B(i,j) < \mathcal{D}_X(i,j)$ . (Hunsberger 2001)



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### Winner Determination \*

- Modify existing WD algorithm (Sandholm 2002) to accommodate temporal constraints.
- Depth-first search in space of partial bid-sets
- Maintain STN,  $(\mathcal{T}_X, \mathcal{C}_X \cup \mathcal{C}_B)$ , containing constraints from proposed activity plus those from bids currently being considered.
- Backtrack if this STN becomes inconsistent.
- \* (Hunsberger & Grosz 2000)
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## Post-Auction Coordination

- Auction yields viable allocation of tasks, but typically results in temporal dependencies among tasks being done by different agents.
- Solution 1: *Temporally decouple* the task-sets being done by different agents (adds constraints, but no need for subsequent coord'n.).
- Solution 2: Relative Temporal Decoupling (weaker constraints, but requires some subsequent coordination).

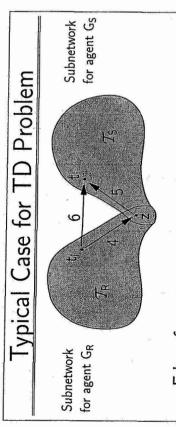
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## Temporal Decoupling (TD)\*

- Goal: Enable agents to operate independently —and hence without communication.
- Method: Add new constraints to ensure mergeable solutions property.
- Will focus on two-agent case, but works for arbitrarily many agents.

(Hunsberger 2002a; 2002b)

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- Edge from  $t_i$  to  $t_j$  not dominated by a path through z.
- Can fix by strengthening edge from  $t_i$  to z, or edge from z to' $t_j$ , or both.

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### TD Algorithm\*

- Add *intra*-subnetwork constraints to ensure that each tight, proper, *inter*-subnetwork constraint is dominated by a path through z.
- Requires processing each such edge only once.
- Afterward, no matter how each agent tightens constraints in its own subnetwork, all intersubnetwork constraints will be satisfied.

(Hunsberger 2002b)

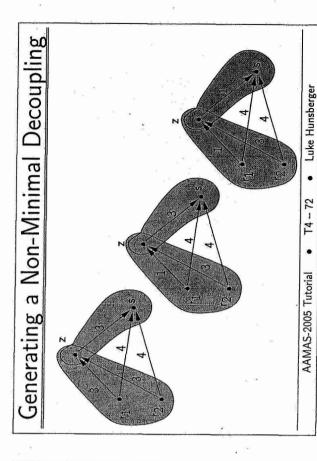
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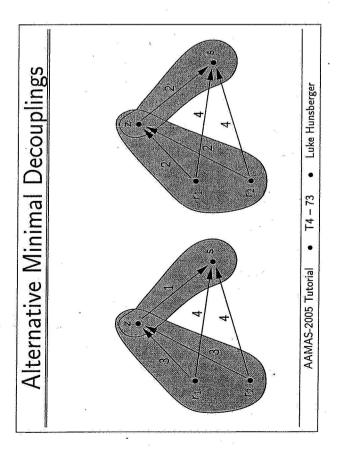
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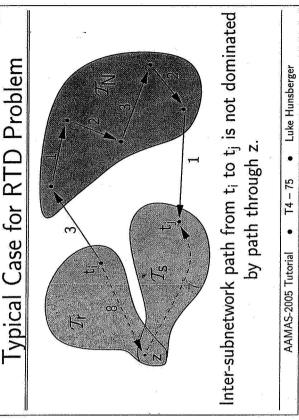
## Improvements to TD Algorithm

- When selecting inter-subnetwork edges to work on, and when deciding how much to tighten each intra-subnetwork edge, use heuristics to increase flexibility in resultant decoupled subnetworks.
- Use *Iterative Weakening* algorithm to ensure minimal temporal decoupling (i.e., one in which any further weakening would foil the decoupling).

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# Relative Temporal Decoupling (RTD)\*

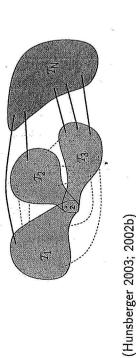
Goal: Use weaker constraints, but allow some inter-subnetwork dependence to remain.

(1) Replace each tight, proper, inter-subnetwork

path by an explicit edge

RTD Algorithm\*

Method: Given N subnetworks, (N-1) are fully decoupled; but  $N^{th}$  dependent on the rest.



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(2) Run TD algorithm ignoring N<sup>th</sup> subnetwork.

(Hunsberger 2003; 2002b)

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### Lambda Bounds for RTD\*

- After RTD, agent controlling  $N^{th}$  subnetwork is dependent on the rest.
- Must not re-introduce any inter-subnetwork paths that would threaten the RTD. (Requirements captured in *Lambda Bounds*.)
- Unlike other agents,  $N^{th}$  agent may add edges linking  $N^{th}$  subnetwork with other subnetworks.

(Hunsberger 2003; 2002b)

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### Other Applications of RTD

- Submitting a bid imposes restrictions on the bidder that are precisely captured by the Lambda Bounds (where N=2).
- The RTD algorithm may be recursively applied yielding an arbitrarily complex hierarchy of dependence and independence.
- Hadad et al. (2003) present an alternative approach to temporal reasoning in the context of collaboration.

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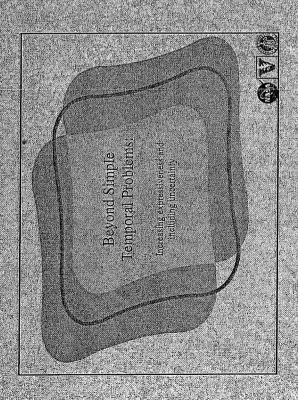
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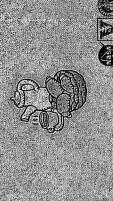
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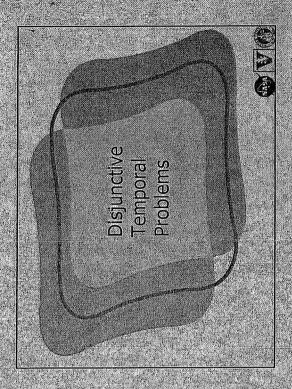
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### The Breakfast Plan (Version 2)

Prepare coffee and toast. Have them ready within 2 minutes of each other. Brew.coffee for 3-5 minutes; toast bread for 2-4 minutes.





### Expressiveness and Uncertainty

- Increasing the expressiveness of the temporal constraints
  - Definition Disjunctive Temporal Problem

    - Solving DTPs Dispatching DTPs
- Planning with temporal constraints
- Explicitly representing uncertaints





### Real Plans often have Disjunctive Constraints

Typical Plan for an Autominder User

ACTION	TARGET TIME
Start laundry	Before 10 a.m.
Put clothes in dryer	Within 20 minutes of washer ending
Fold alothes	Within 20 minutes of dryen ending
Prepare lunch	Between 11:45 and 12:15
-Bat lunch	At end of prepare lunch
Gheck pulse	Between 11:00 and 12:00 rand between 3:00 and 4:00
Depending on pulse,	At end of check pulse

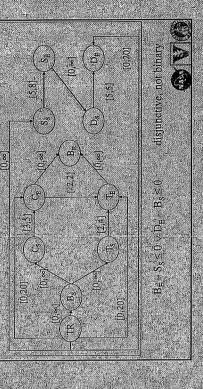
Prepare coffee and toast. Have them ready within 2

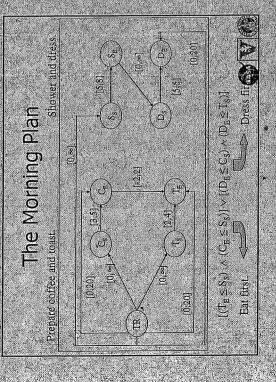
minutes of each other. Brew coffee for 3-5

The <del>Breakfast</del> Plan (Version 3) Morning

### minutes, toast bread for 2-4 minutes. Also take a shower for 5-8 minutes, and get dressed, which akes 5 minutes. Be ready to go by 8,20

### The Morning Plan





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# AAMAS2005 - T4 - Temporal and Resource Reasoning for Planning, Scheduling and Execution in Autonomous Agents

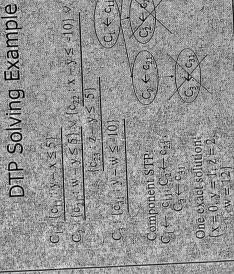
### Disjunctive Constraints

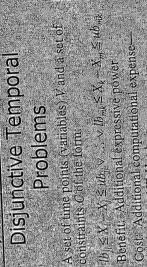
- Represent non-overlaps (as in our example)
   Can also represent other forms of disjunction
  - E.g., take a shower for 5 minutes or a bath for 10 minutes.



### DTPs as CSPs

- .. One-Level Approach. - Direct assignment of times to DTP variables:
- = Limitations: difficult to deal with infinite domains: produces overconstrained solution
  - Two-Level Approach
- Construct a meta-level CSP - Variables! DTP constraints
- Domains. Disjuncts from DTP constraints
- Constraints: Implicit assignment must lead to a "consistent component STP"





reasoning is NP-Hard

reasoning is NP-Hard

True even for binary problems, i.e., constraints have the form

 $\|b_{jj} \leq X + Y \leq ab_{ji} \nabla \cdot \cdot \cdot \wedge 1b_{mk} \leq X + Y \leq ab_{mi}$ 



## Strategies for Efficiency

Removal of Subsumed Variables

- Forward checking / incremental forward checking
  - Conflict-directed backjumping
- Removal of subsumed variables

partial as

- Semantic branching
  - · No-good learning
- . Use efficient SAT solvers for meta-level

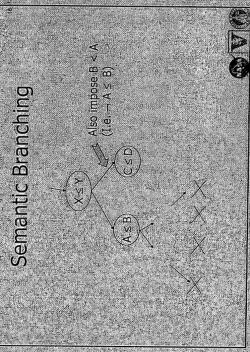


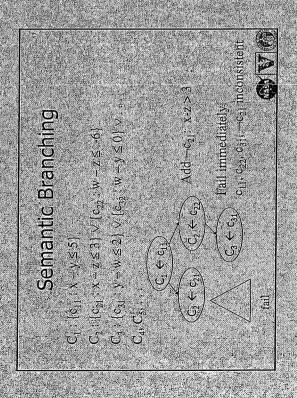
### Removal of Subsumed Variables

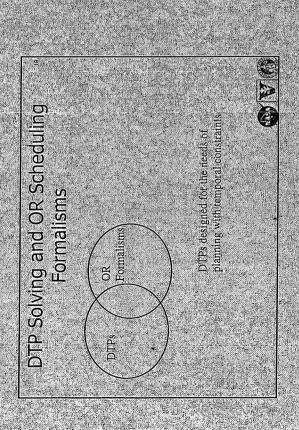
$$\begin{split} &C_1: \{c_{11} : y - x \le 5\} \\ &C_2: \{c_{21} : x + z \le 5\} \lor \{c_{22} : w - \bar{y} \le \pm 10\} \\ &C_3: \{c_{31} : y - z \le 15\} \lor \{c_{32} : z - v \le 10\} \lor \end{split}$$

 $\mathbb{C}_{\mathbf{i}} \leftarrow c_{\mathbf{i}\mathbf{i}}$ 

c<sub>11</sub> and c<sub>21</sub> imply c<sub>31</sub>; so no need to try other values for C<sub>3</sub> along—this branch ું ≮ુલા

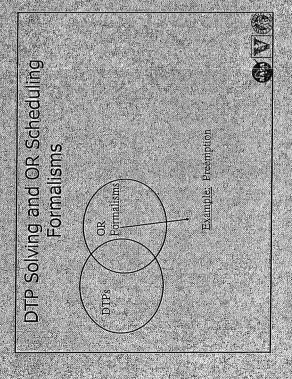








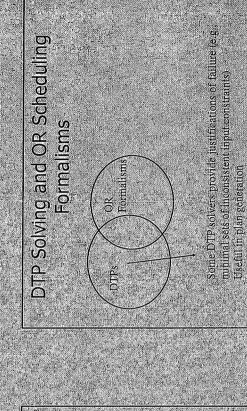
DTP Solving and OR Scheduling Formalisms

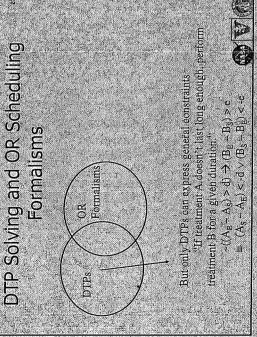


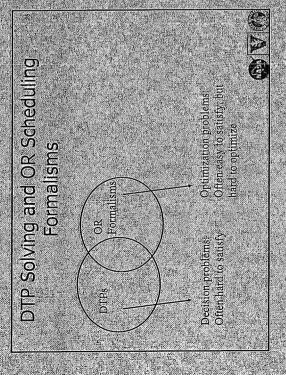
. ISS & DTP can both express non-overlap constraints:  $A < B \vee B < A \ (\mbox{binary with intervals (GaSks), non-$ 

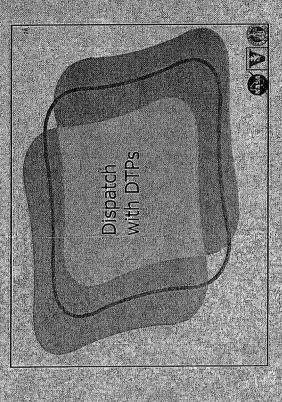
binary with time points

Bxample: Arbitrary Disjunction









### DTP Dispatch Method #1

- · With total control of the execution process:
- Given a DTP, find a consistent component STP 3
- Dispatch Susing STP dispatch algorithm

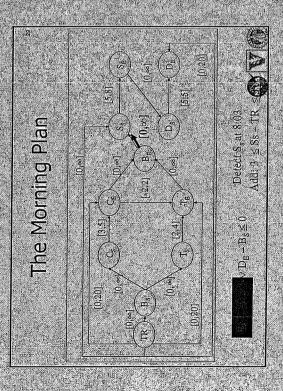


### DTP Dispatch Method #2

- With partial control of the execution process (e.g., in execution monitoring)
- Given a DTP, find a consistent component STP S. While no events inconsistent with S occur - Dispatch Susing STP dispatch algorithm
- Otherwise, if event e occurs at time t that is inconsistent with S - Add an execution constraint,  $t \le s - TR \le t$

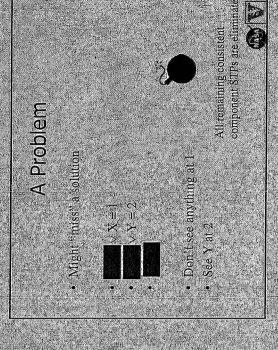
- Find a new consistent component STP S





### DTP Dispatch Method #3

- Specifies what actions are live and enabled (what can be done)
- An event e in a DTP is live iff now is in its time window
- An event e in a DTP is enabled ifficis enabled in ai least one consistent component STP
- And what must be done.
  - Deadline Formula
- Specifies what deadline must be sausfied next (what must be done)



#### Example

$$\begin{split} & G_{1} \cdot [c_{11}, \cdot 5 \le x + TR \le 10] \cdot v \cdot (c_{12}, \cdot 15 \le x + TR \le 20) \\ & G_{2} \cdot (c_{21}, \cdot 5 \le y + TR \le 10) \cdot v \cdot (c_{22}, \cdot 15 \le y + TR \le 20) \\ & G_{3} \cdot (c_{31}, \cdot 6 \le x + y \le \infty) / v \cdot (c_{32}, \cdot 6 \le y + x \le \infty) \\ & C_{4} \cdot (c_{41}, \cdot 11 \le z + TR \le 12) \cdot v \cdot (c_{42}, \cdot 21 \le z + TR \le 22) \end{split}$$

Consistent Component STPs:

- .1. STP1:  $g_{11}, g_{21}, g_{31}, g_{41}$  × before y, z early  $2^{-1}$  STP2:  $g_{11}, g_{23}, g_{31}, g_{42}$  × before y, z late



#### Example

 $\begin{aligned} & \mathbb{C}[n_1(z_{11}, 5 \leq x + \mathrm{TR} \leq 10)] \times (z_{12}, 15 \leq x + \mathrm{TR} \leq 20), \\ & \mathbb{C}_{21}(z_{21}, 5 \leq y + \mathrm{TR} \leq 10)) \times (z_{21}, 15 \leq y + \mathrm{TR} \leq 20), \\ & \mathbb{C}_{31}(z_{21}, 6 \leq x + y \leq \infty) \times (z_{22}, 6 \leq y + \lambda \leq \infty), \\ & \mathbb{C}_{41}(z_{41}, 11 \leq z + \mathrm{TR} \leq 12) \times (z_{42}, 21 \leq z + \mathrm{TR} \leq 22), \end{aligned}$ 

Execution Table:

<x, ([5,10], [15,20])>,<y, ([5,10], [15,20])>

Enabled events and their lime windows

- Deadline Formula:

<x < y, 10>

CNF formula that must be satisfied "next"



## Generating the Deadline Formula

Generate-DF (Solutions, STE [1])

Let U = the set of upper bounds on time windows,  $U(x_i)$ , for each still unexecuted action x and  $e_{\underline{a}}$  of  $\overline{S}$   $\overline{F}$   $\overline{F}$   $\overline{F}$ 

Let NG, the next ortical time point, be the value of the minimum bound in U.

 $\operatorname{Let}^{\bullet}\operatorname{U}_{MIN} \neq [\operatorname{U}(x_{i}, i)] \operatorname{U}(x_{i} i) = \operatorname{NG}],$ 

. For each x such that  $U(x,i)\in U_{MN}$ , let  $S_x=\{1\}$   $U(x,i)\in U_{MN}$ . Initialize  $F=true_i$ 

For each minimal solution MinCover of the set-cover problem (Solutions ,  $\cup S_1$ ) let  $F = F \times (v \times \mid S_1 \in MinCover N)$ . Output  $DF = \langle F, NC \rangle$ .



### Dispatch Method

Computing the Execution Table:

Find all enabled events

-:Compute their time windows in every consistent component STP

Computing the Deadline Formula:

# Phid the next time at which some event must occur — Find all events that might have to occur by that time  Compute the minimal event sets that would ensure that not all remaining consistent component STPs are eliminated.



## Generating the Deadline Formula

Generate-DF (Solutions, STP [1])

Let  $U = \text{the set of tupper bounds on time windows, } \overline{U}(xa)$  for each still unexecuted action x and each STP i.

Let NG, the next critical time point, be the value of the minimum upper bound in U

Let  $\mathbb{U}_{Min} = \{ \mathbb{U}(x_{i,i}) | \mathbb{U}(x_{i}) = \mathbb{N}\mathbb{C} \}$ 

For each x such that  $U(x,t) \in U_{Kirk}$  let  $S_x = \{i \mid U(x,t) \in U_{Kirk}\}$ . Initialize  $P = true_i$ .

For each minimal solution MinCover of the sereover problem (Solutions,  $-S_1$ ), let  $F = F \times (\sqrt{x} \mid S_1 \in MinCover x)$ . Output  $DF = \langle F, NC \rangle$ 



#### Example

 $\{c41; 11 \le z + TR \le 12\} \lor \{c42; 24 \le z + TR \le 22\}$ Cli (cll): 5≤x−TR≤10] » (cl2: 15≤x−TR≤20) C2: (c21::5≤y−TR≤10) » (c22: 15≤y−TR≤20) G3:  $(c31: 6 \le x - y \le \infty) \lor (c32: 6 \le y - x \le \infty)$ G4:  $(c41: 11 \le z - TR \le 12) \lor (c42: 21 \le z - TR$ 

Consistent Component STPs: STP2; c11, c22, c32, c42 STP3; e12, c21, e31, c41 STP1: c11, c22, c32, c41 STP4; c12, c21, c31, c42

U(x, 1) = U(x, 2) = 10 U(x, 3) = U(x, 4) = 20 U(y, 1) = U(y, 2) = 20(z,1) = U(z,3) = 12(y,3) = U(y,4) = 10



Output DF = <F, NC>

#### For each minimal solution MinCover of the set-cover problem (Solutions, $\cup S_1$ ), let $F = F \times (v \times I) \setminus S_k \in MinCover \times I$ For Each x such that $U(x,y)\in U_{kitN}$ let $S_k=\{l\mid U(x,t)\in U_{kitN}\}$ Let $\overline{U}_{MIN} = \{\overline{U}(x,t)|\overline{U}(x,t)=NG\}$ . upper bound in U. Initialize F = true;

Jet NG: the next critical time point, be the value of the minimum

Let U = the set of upper bounds on time windows, U(x,i) for

Generate-DF (Solutions: STP [1])

each still unexecuted action x and each STP1

Generating the Deadline Formula

## Generating the Deadline Formula

Let NC, the next critical time point be the value of the minimum upper bound in U

For each x such that  $U(x,i) \in U_{Min}$ , let  $S_x = (1 \mid U(x,i) \in U_{Min})$ Initialize F = (rue;

Output  $DP = \langle E, NC \rangle$ 

 $\mathbf{U}_{MiN} = \{(x,1), (x,2), (y,3), (y,4)\}$ 

U(y,1) = U(y,2) = 20 U(y,3) = U(y,4) = 10

U(z,1) = U(z,3) = 12 U(z,2) = U(z,4) = 22

STP4; 612, 621, 631, 642

STP3; c12, c21, c31, c41

U(x,1) = U(x,2) = 10 U(x,3) = U(x,4) = 20

Consistent Component STPs:

STP1; c11, c22, c32, c41 STP21:c11, c22, c32, c42



3

Generate-DF (Solutions: STP [il])

Let  $U \equiv the$  set of upper bounds on time windows, U(x,t) for each still unexecuted action x and each STP i

C3: [c31: 6≤x-y≤∞]∨ {e32: 6≤y-x≤∞] C4: (c41: 11 ≤z-TR≤12) ∨ (c42: 21 ≤z-TR≤22)

(eith  $5 \le x - TR \le 10$ )  $\lor$  (eith  $15 \le x - TR \le 20$ ) C2:  $\{c21\}, 5 \le y - TR \le 10\} \lor \{c22 - 15 \le y - TR \le 20\}$ 

Example

Let  $\mathbf{U}_{MN} = \{\mathbf{U}(\mathbf{x}, \mathbf{i}) | \mathbf{U}(\mathbf{x}, \mathbf{i}) = \mathbf{NC} \}$ .

For each minimal solution MinGover of the set-cover problem (Solutions,  $\cup S_s$ ), let  $F = F \wedge (\vee \times) S_s \in MinCoverx$ ).

#### Example

GI:  $|c_{11}|$ ,  $5 \le x + TR \le 10$ )  $\sim |c_{12}|$ ,  $|c_{23}| \le x + TR \le 20$ ];  $|c_{21}|$ ,  $5 \le y + TR \le 10$ ]  $\sim |c_{22}|$ ,  $|c_{23}| \le y + TR \le 20$ ]  $|c_{23}|$ ;  $|c_{23}|$ ; |

Consistent Component STPs.
STP1: 611, 622, 632, 641
STP2: 611, 622, 632, 642
STP3: 612, 621, 631, 641
STP4: 612, 621, 631, 642

 $\begin{aligned} & \text{NC} = 10 \\ & \text{Usin} = \{(x,1), (x,2), (y,3), (y,4)\} \\ & \text{S}_{x} = \{1,2\} \\ & \text{S}_{y} = \{3,4\} \end{aligned}$ 



#### Example

Consistent Component STPS STP1: c11, c22, c32, c41 STP2: c11, c22, c32, c42 STP3: c12, c21, c31, c41 STP4: c12, c21, c31, c42

 $S_{i, = \{1, 2\}}$   $S_{i, = \{3, 4\}}$  $MSG(\{1, 2, 3, 4\}, \{S_{i, 1}\}, \{S_{i, 1}\}) = \{\{S_{i, 1}, S_{i, 1}\}\}$ 



## Generating the Deadline Formula

Generate: DF (Solutions, STP [il])

Let U= the set of upper bounds on time windows,  $U(\vec{x}\vec{n})$  for each still unexecuted action  $\vec{x}$  and each STP U

Let NG, the next citical time point, be the value of the minimum upper bound in U

Let  $U_{MiN} = \{U(x, 1) | U(x, 1) = NC\}$ .

For each x such that  $U(x_1)\in U_{M(N)}$  let  $S_x=\{i||U(x_0)|\in U_{M(N)}\}$  Initialize F=(iv)

For each minimal solution MinCover of the set-cover problem (Solutions  $\mathbb{T} \cup S$ , ), let  $F = F \setminus A(\times X) \mid S_x \in MinCover(x)$ . Output  $DF = \times F \setminus N(S_x)$ 



### Larger Deadline Formula

Suppose

=4 consistent component STPs  $\pm NC = 10$ 

-1, U(x, 1) = U(x, 2) = U(y, 3) = U(y, 4) = U(z, 4) = U(x, 3) = U(x, 3) = U(x, 4) = U(x, 4) = U(x, 3) = U(x, 4) =

The minimal set covers are

- (S., S.) and (S., S., S.). So the deadline formula is

(W > 2 > X) > (X > X) +



35

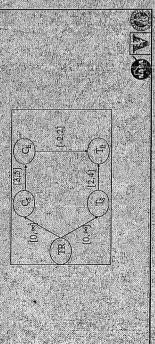
### The Dispatch Bottleneck

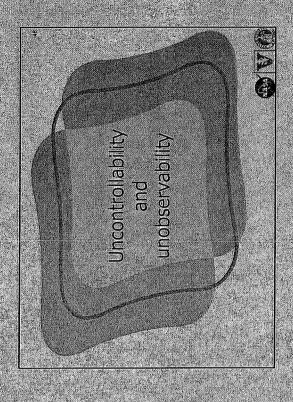
- Requires computation of all component STPs
  - May be exponentially many of them
- Open Research Question: Can we identify "representative" sets of component STPs?



### Breakfast Again

 You don't really get to control how long the coffee brews (but you can pop the toast at any time).





### Handling Temporal Uncertainty

- TP-u (e:g., STP-u)
- Distinguish between two kinds of events:

   Controllable: the executing agent controls the time of
- Uncontrollable: "nature" controls the time of occurrence



Controllable edge (Y controllable event)



. Uncontrollable edge (Y uncontrollable event)



# Three Notions of "Solution"

- Strongly Controllable: There is an assignment of time points to the controllable events such that the constraints will be satisfied regardless of when the uncontrollables occur.
- One (or more) solutions that work no matter what!

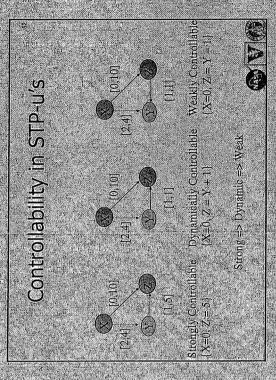


# Three Notions of "Solution"

- Dynamically Controllable: As time progresses and uncontrollables occur, assignments can be made to the controllables such that the constraints are
- Solutions that are guaranteed to work can be created conditionally to observations.



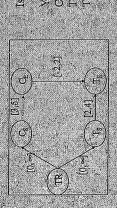
# Three Notions of "Solution" • Weakly Controllable: For each outcome of the uncontrollables, there is an assignment of time points to the controllables such that the constraints are satisfied. • One (or more) solutions that work for each outcome.



37

## Breakfast Again

You don't really get to control how long the coffee. brews (but you can pop the toast at any time):



Ts treontrollable?



# Controllability and Dispatchability.

- Controllability: defines policies to determine times for controllable events depending on knowledge of throontrollable events occurrence.
- Dispatchability, identifies effective propagation paths such that knowledge on the execution of an event constrains the possible execution times for other events



# Controllability and Observability

- Different notions of controllability make different assumptions about what can be observed
- Strong Controllability, incontrollable events cannot be observed and consistency must be guaranteed
- Dynamic Controllability: uncontrollable events can be observed and consistency must be guaranteed Weak Controllability. "I'm feeling lucky". and
  - Weak Controllability: "I'm feeling lucky"... and luck will always be in a position to help achieve consistency



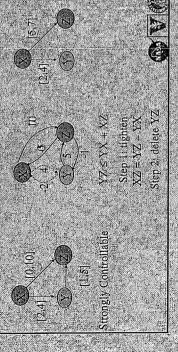
## Execution Policies

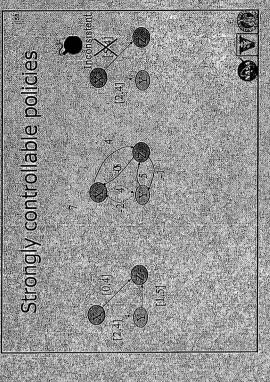
- Controllability definition emphasizes existence of solutions
- At execution time we need policies to make decision as a function of our knowledge
- Like in the case of STPs, provide ways to determine bounds and repropagation methods to create solutions on the fly



# Strongly controllable policies

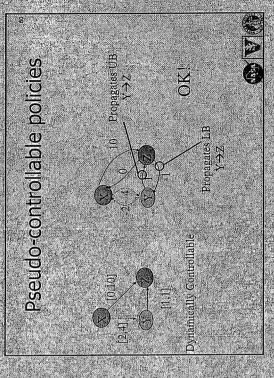
 We need to come up with policies assuming no knowledge about the uncortrollable event
 Solution disconnect any dispatchable tink from the event



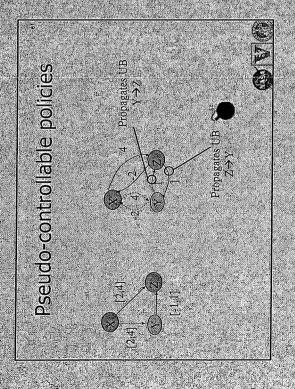


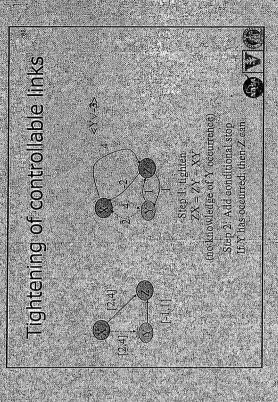
# Pseudo-Controllability

- The upper and lower bounds of an uncontrollable event are not necessarily propagated outside of the uncontrollable link (no necessary tightening of uncontrollable links).
  - Bound propagation can originate from an uncontrollable event because we can have knowledge of its occurrence...
- ... but during execution there can be executions that propagate into the uncontrollable event tighter bounds than the uncontrollable link (possible tightening of the uncontrollable links) ©



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## Wait Propagation Rules

- "Wattlinks" are a new type of "partally uncontrollable" link. If they are present, they cause execution to be contingent on t
- Unike uncontrollable links, they can be eliminated through tightening



Computing Dynamic

Controllability of an STPU

• Use triangular reductions

• Case 1: v < 0.

= Briellow C, so de

• Case 2: u ≥ 0.

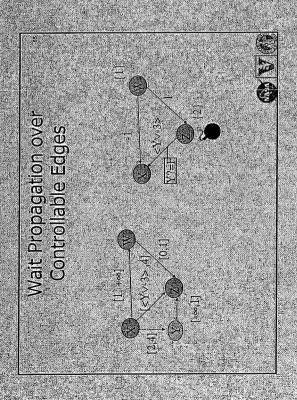
= Briesach C: tighten AB to fy-v, (x,y)

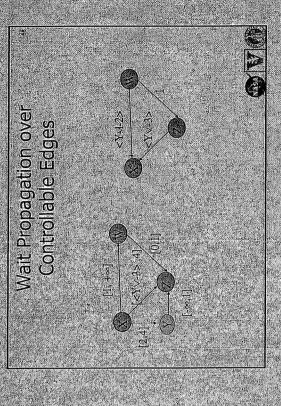
xsul to make de

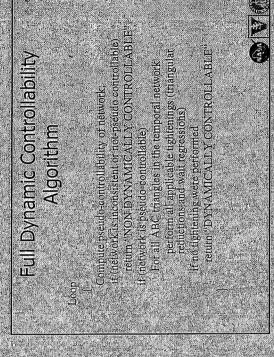
• Gase 3: u < 0 and v ≥ 0.

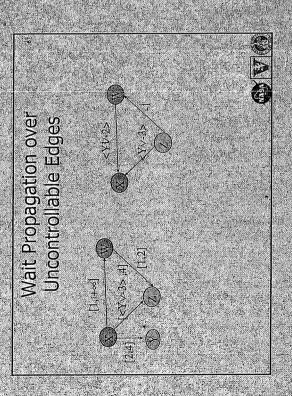
- B is unordered win! C. tighten
lower bound of AB to (Cory-v) to make de.

- B is unordered win! C. tighten
lower bound of AB to (Cory-v) to make de.









3

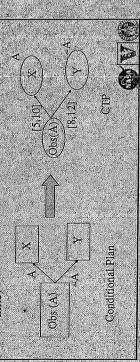
## **Termination Condition**

- Without further analysis, the algorithm is pseudo-polynomial
  - Pseudo-controllability: O(NE +  $N^2 log N$ )
    - Tightening: O(N3)
- Number of repetition of cycle: U, number of time units in widest time bound
  - Complexity: O(U N3)
- U could be very large



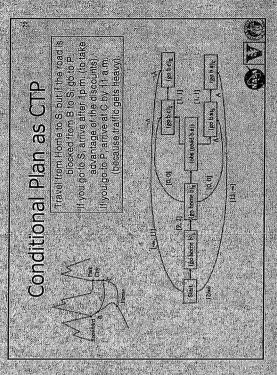
# Handling Causal Uncertainty

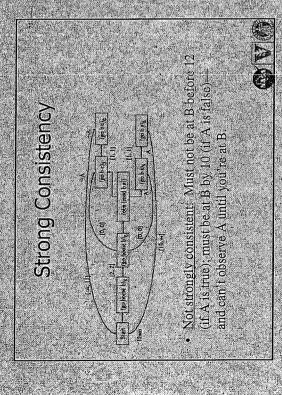
- CTP (e.g., CSTP)
- Label each node—events are executed only if their associated label is true (at a specified observation time)

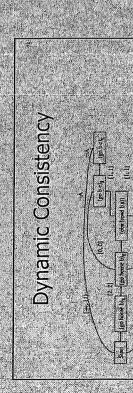


### Cutoff bound

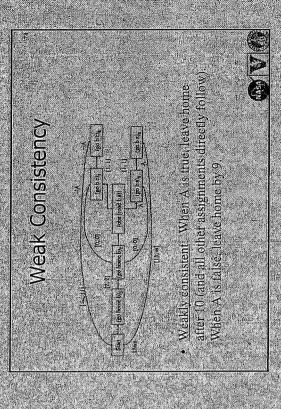
- Since the number of edges is finite, indefinite jughtening is due to the existence of propagation cycles
- Cycle traversal must repeat after a maximum number of propagation (as in the Bellman-Ford algorithm for shortest paths
- Cutoff bound for dynamic controllability:
   E. O(NK) with K = number of non-controllable links.
  - Cutoff on the number of cycles gives O(KN4) complexity bound.

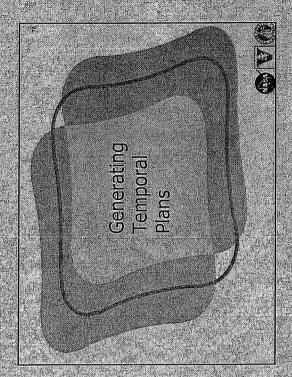






- Not dynamically consistent: Can't tell when you need to leave home until it's too late.
- Variant that is is dynamically consistent: Add a parking lot at B where you can wait.





## Generating Temporal Plans

- Various models have been developed, dating back to the early 1980's (DEVISER)
- Beginning to see a convergence in the Constraint. Based Interval approach
  - Model the world with
- Attributes (features): e.g., coffee
- Values that hold over intervals: e.g., brewing
- = Times points that bound the intervals: e.g., b, be
  - Axioms that relate the values



# Temporally Quantified Assertions

- Each feature takes a single value at a time, i.e. formally there are a set of functions fi(feature, time,) → value, where value ge domain (feature)
- Temporally qualified assertions (tga's or just "assertions")
  - holds (toaster-content, X, Y, empty) holds (coffee, 8:03, 8:05, brewing)
- holds(F,s,e,P)  $\wedge$  holds(F,s',e',Q)  $\rightarrow$ [e<s' v e' <s v P=0] Uniqueness Constraints!



## Features and Values

none, brewing, ready, stale untoāsted, toasting, toast Domain of Values at(X), going(X,Y)no, dressing, yes. empty, full yes, no on, off ves, no yes, no Foaster-Contents Joaster-Status Showering Bathing Location Dressed Bread Clean



## Planning Axioms

- Used to model actions
- Basic form

Effect →

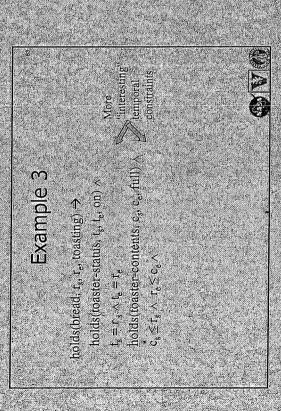
Action 2 A Preconditions 2 A Constraints,) w (Action 1  $\wedge$  Preconditions  $_1$   $\wedge$  Constraints,)  $\vee$ 

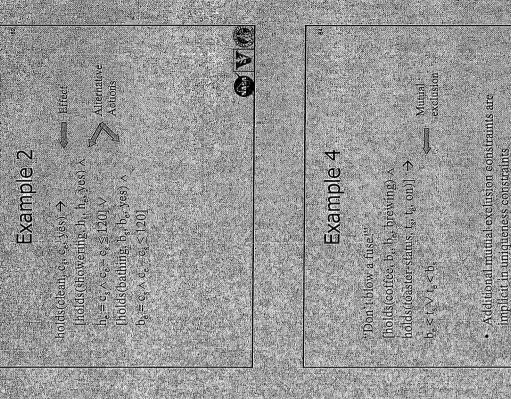
(Action no Preconditions, o Constraints,

- · Can also partition the knowledge differently
- · And can also use axioms to model other types of constraints (e.g., mutual exclusion)

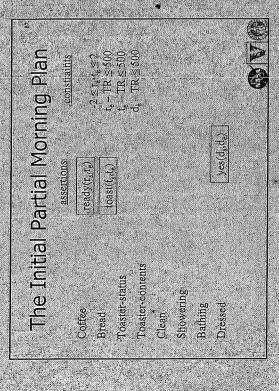


# Example 1 holds(coffee, r., r., ready) → ← Effect holds(coffee, b., b., brewing) ∧ ← Action (be = r.) ∧ (3 ≤ be = b. ≤ 5) ← Addil Constraints holds(coffee, n., n., none) ∧ ← Addil Constraints ne = bs Can also split out into two axioms Effect → Action Action → Preconditions





#### 



## The Planning Problem

- Given a set of features and their domain, a (partial)
  - <u>plants</u>
- a set of assertions on those features and
   a set of constraints on the time points of the assertions
- A solution is
- = a complete assignment of values to features
  - such that all of the constraints are satisfied



## Expanding a Plan

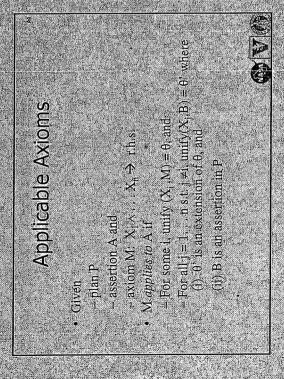
- Select an assertion
- Find all the axioms that apply to it
  - For each of those axioms

- Cheose an alternative (one disjunct in the tail of the axiom)

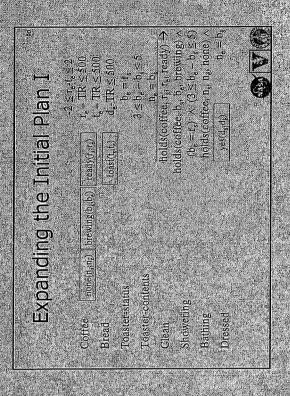
- Ensure that the assertions and constraints in the chosen disjunct are in the plan, either by adding them or unifying them with assertions and constraints already present

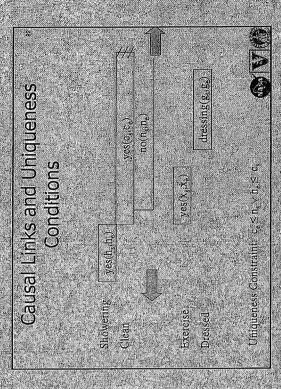


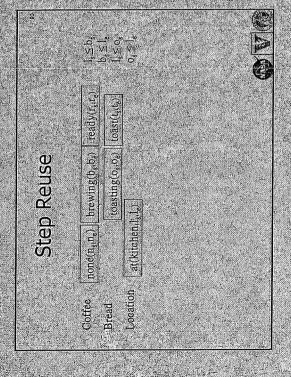
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## CBI Planning Algorithm

Unchecked, Assertions & initial assertions

Expand (Unchecked, Assertions, Constraints, Axioms)

If Constraints are inconsistent, fail.

If Unchecked = Ø, return <Assertions, Constraints>

Select u & Unchecked

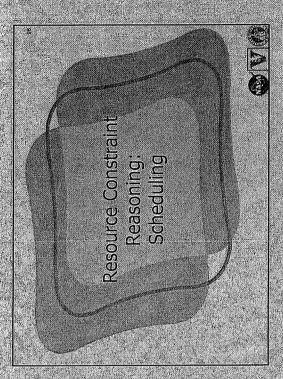
For every axiom X & Axioms that applies to u

Choose an alternative of from X (dis the result of the unification that equess X to be applicable)

Add constraints 6 e d to Constraints Expand(Unchecked, Assettions, Constraints, Axioms)

New: Add s to Assertions and Unchecked

# Underlying Constraint Network The temporal constraints form a DTP Technically, a dynamic DTP, since time points are added incrementally Use DTP rechniques to check consistency efficiently



#### Outline

- Resource representations
- Relationship between planning and scheduling representations
- Search spaces: flexible plans and fixed time instantiations
- Resource contention measures
  - = Probabilistic | Courae/unnar.hou
- Lower/upper bounds
  - Envelopes



### Operating the stove The Planning Perspective

7x = {cpot, pan}

clear(stove)

del 
clear (stove)

on  $(7x_i, stove)$   $\frac{pre}{takeOff (7x_i, stove)}$  del on  $(9x_i, stove)$ 

10 A C

Freakfast at Yosemite

Tourish a stove with just one burner.

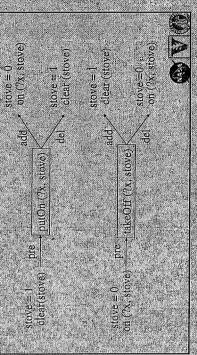
The stopping of the stopping so you cook the toast on a pan.

The stopping so you cook the toast on a pan.

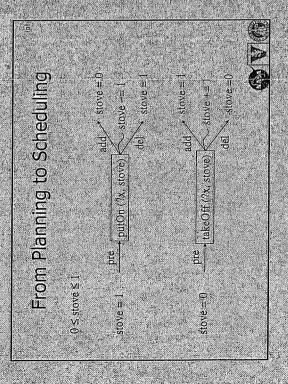
The stopping so you cook the toast on a pan.

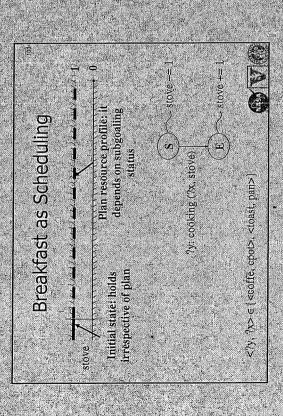
The stopping so you cook the toast on a pan.

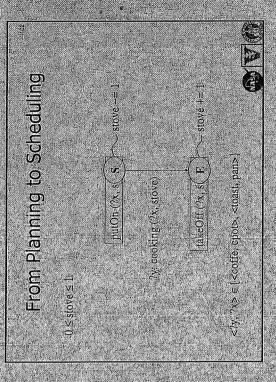
# - From Planning to Scheduling



20





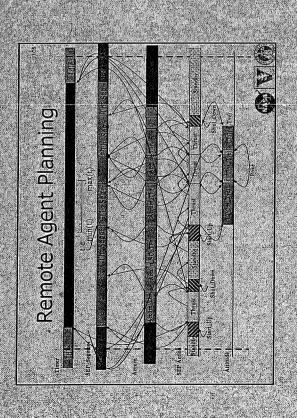


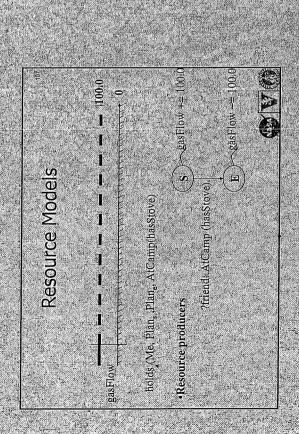
## A View of Planning and Scheduling

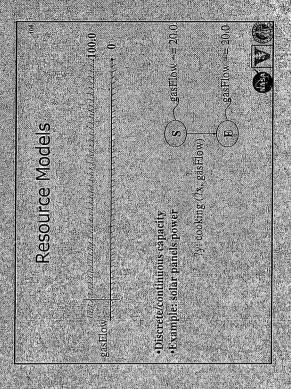
- Planning primarily focuses on constructing a consistent evolution of the world (states and transitions).
- Scheduling almost entirely focuses on handling mutual exclusion and deadlines
- . . but since the beginning planning was also addressing scheduling flaws can be often seen as scheduling conflicts
  - Graphplan and mutual exclusions implicitly brought this concept to the forefront

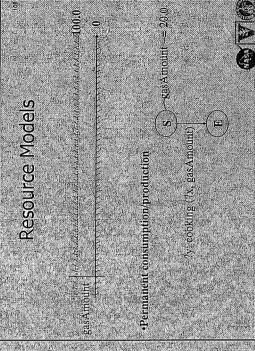


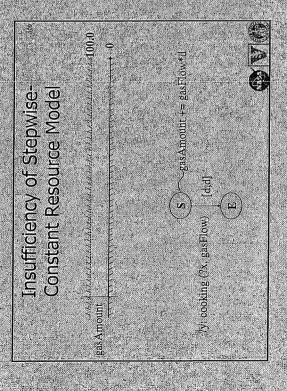
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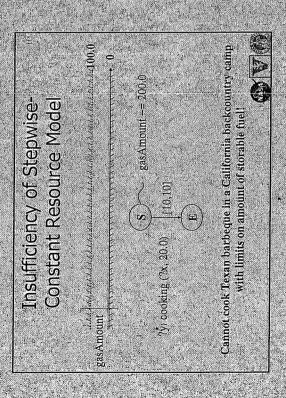


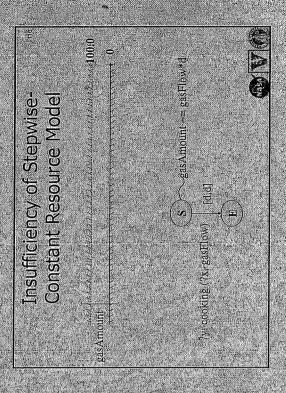


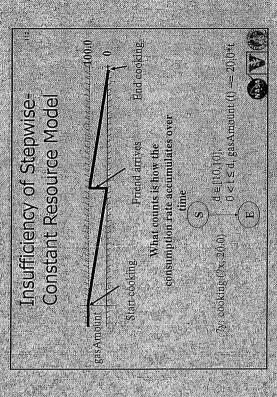












57

# Flexibility in Plans/Schedules

How to Build a Flexible Breakfast

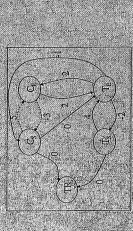
Schedule

- Not a lot-is happening in the vacuum of space, though Fundamental obstacles in the real world
- Two possible strategies + Flexible policies



# How to build a flexible schedule

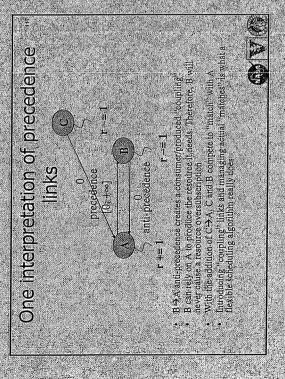
How to build a flexible schedule

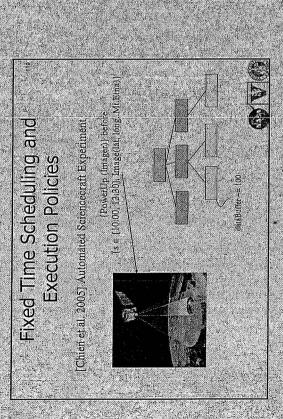


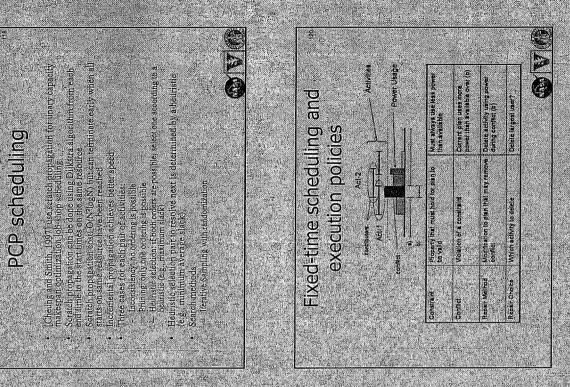
Can we start brewing the coffee after the toast is ready?

53

Can we start making the toast after the coffee is brewed? YES







K

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# Conflict Repair Methods

- Use a repair method to eliminate a conflict
- ASE uses a planner, not just a scheduler.
- Hence it is possible to generate new activities or select different task decompositions
- Repair methods
- handled in classical resource, Not Add producer of - move an activity → delete an activity
  - detailing an activity - add a new activity

scheduling

- abstracting an activity.

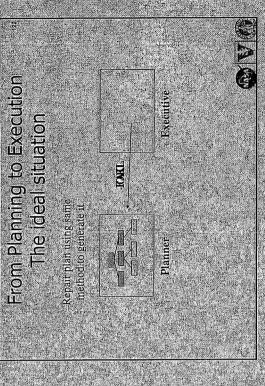


## Comparison of Flexible and Fixed Policies (1)

Simple and intuitive to implement

4 It is easier to think of heuristics based on resource profiles

- · More compact data structures
  - Less costly propagation
- · Plan does not give "declarative" measure of robustness
- Execution repair is fundamental to robustness
- A full plan repair process may be too expensive at
  - ASE has only 4 MIPS available



## Comparison of Flexible and Fixed Policies (2)

- Flexible policies
- Plan guarantees measure of robustness \* - Flexible policies break less often
- · Execution time adjustments are intrinsically fast (propagation vs planning).
- · More complex
- But complexity and computational expenses mostly affect off-line
- Actual value of flexibility is only as good as the semanties of he representation
- ... and this is why you are taking this tutorial



## What actually happens on ASE From Planning to Execution

Building flexible policies from

fixed time schedules.





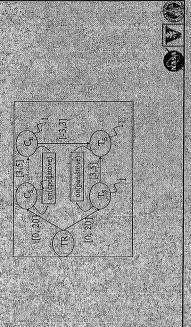
#### Planner

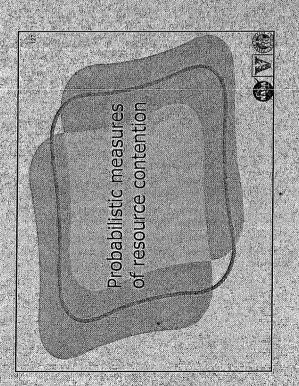
#### Executive





Contentious Breakfast





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# Time bounds and resource



- Without further coordination, C and T are free to collide for the use of the stove
- and eventually eliminate the possibility of conflict ("couplings" of producers to consumers) reduce The inclusion of anti-precedence links



#### **Temporal Information for** Contention Analysis



- Partial temporal information (e.g., time bounds for events) is insufficient to determine informative contention.
  - More (full) temporal information is expensive to acquire and maintain
- There needs to be a balance between cost and utility of temporal/research inferences: Eventual value is in search



## Time bounds and resource conflicts



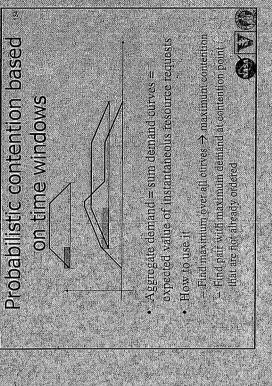
- Without further coordination, C and T are free to collide for the use of the stove
- and eventually eliminate the possibility of conflict ("couplings" of producers to consumers) reduce The inclusion of anti-precedence links.

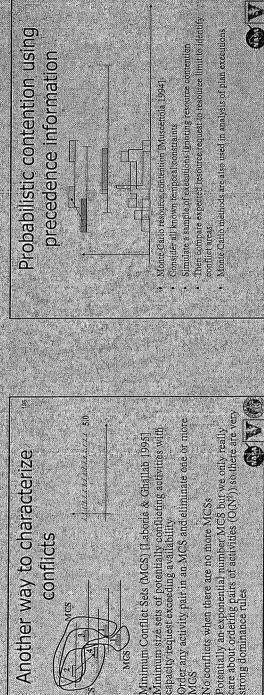
# Probabilistic Resource Contention

- Use probabilistic assumptions to generate time assignments given a temporal network
- Combine probabilistic assignments into contention statistics
- Use contention statistics as the basis for search heuristics
  - Selection of problem sub-structure at the basis of Heuristic factors in probabilistic analysis:
- Probabilistic assumptions on how activities request
- Variable/value ordering rules that use statistics



#### - Fixed durations, consumption at start, same production Probabilistic contention based on Individual action demand inside the time bound $= d_i(t) \equiv \Sigma_{\text{max(est, t-dun)StSmith(IR, t+dun)}} f_i'(III = est)$ time windows [Beck & Fox 2000] Assumptions: - Uniform distribution of start times - Time bounds only





Order any activity pair in an MCS and eliminate one or more

No conflicts when there are no more MCSs

Minimum size sets of potentially conflicting activities with capacity request exceeding availability

Minimum Conflict Sets (MCS) [Laborie & Ghallab 1995]

Another way to characterize

conflicts

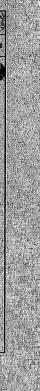
#### Comparison of statistical contention measures

- Monte Carlo simulation is more informed
- Time-window method is less computationall expensive

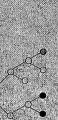
Resource Usage Bounds

- 4 Time windows: O(N) in time and space A Monte Carlo: with sample size S
- . O(S E) in time (if network is dispatchable)
  - . O(S N) in space
- Monte Carlo method also biases sample depending on stochastic full used to simulate the network
  - ÷ ... but the rule can increase realism if it accurately describes execution conditions



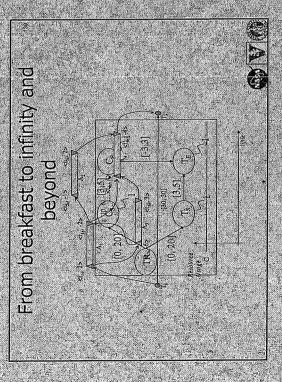


Search Guidance



- The ability of detecting early that the flexible plan is resource/time inconsistent can save exponential amount of work
- Same for early detection of a solution





# Need for exact resource bounds

- Statistical methods of resource contention give sufficient conditions to determine that a solution has not been achieved
- They cannot guarantee either inconsistency or achievement of a solution
- Exact resource bounds can



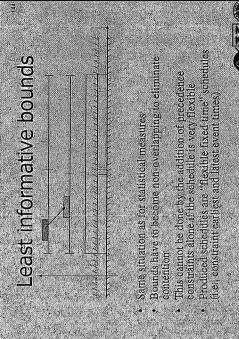
## Bounds are costly

- In summary, bounds try to summarize the status of an exponential number of schedules
  - As in the case of probabilistic measures, we can obtain different bounds depending of how much structural information on producer/consumer coupling we use
- The more information, the tighter the bound
- · The more information, the more costly the bound

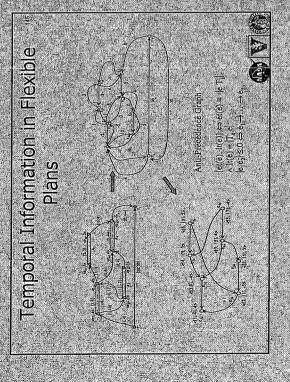


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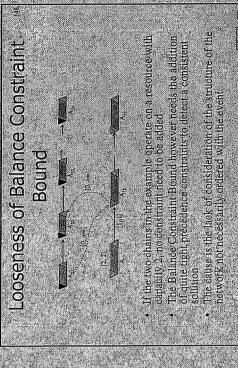
AAMAS2005 - T4 - Temporal and Resource Reasoning for Planning, Scheduling and Execution in Autonomous Agents

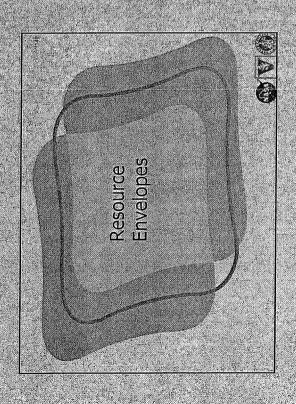


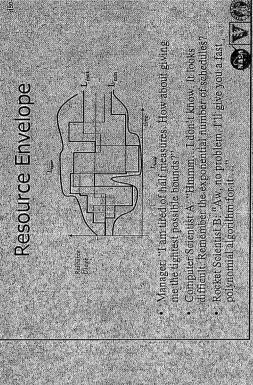
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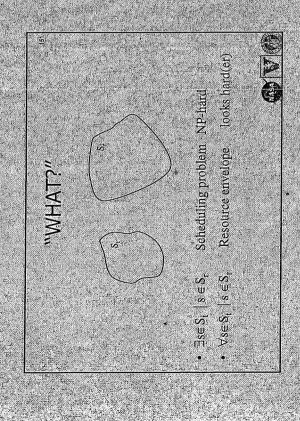
# Cost of balance constraint bound

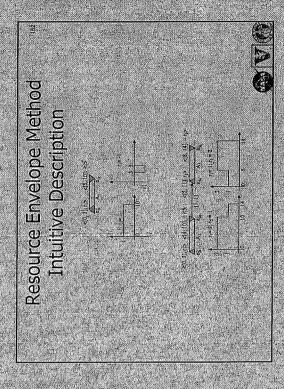
- Non incremental cost (compute the bound from scratch)
  - Find the anti-precedence network: O(NE) / O(NE + N²log·N)
    - Compute bounds from each event: O(NE) / O(N<sup>2</sup>)
- Total cost (time propagation + bounds): O(NE) / O(NE + N<sup>2</sup> logN).
- Indremental propagation can reduce cost per each iteration
   Treat suppose 6.11, 6.2, partition of the state of
  - Used succesfully for optimal scheduling in [Laborie 2001]



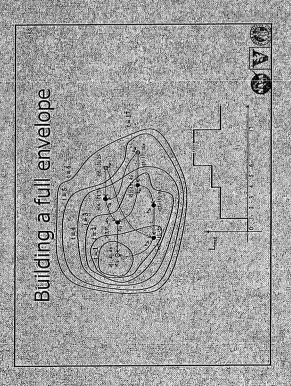


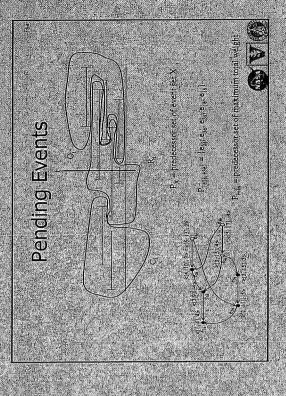






AAMAS2005 - T4 - Temporal and Resource Reasoning for Planning, Scheduling and Execution in Autonomous Agents



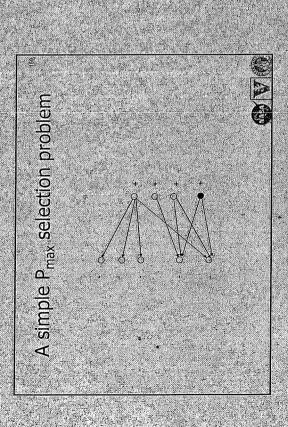


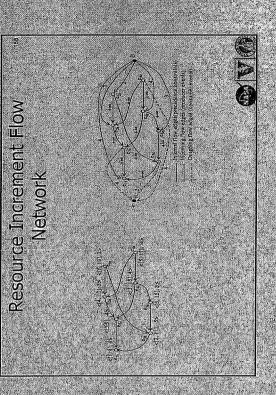
# Maximum flows f(e, e, ) = -f(e, e) f(e, e, ) ≤ (e, e, e) f(e, e, e) ∈ (e, e, e) Algmenting putt = putti front extention Algmenting putt = putti front extention Algmenting putt = putti front extention Algmenting putti = putting front extention Algmenting putting putt

## Key algorithm step

- "Find predecessor set within events that are pending at that causes the maximum envelope increment".
- If we consider all "couplings" (due to anti-precedence links
  posted by the scheduler or due to original requirements);
   we can find sets of events that match. These will balance each other and cause no effect of the envelope level.
  - Events that do not match create a simplify or a deficit
     The amount of surplus (if any) represents the increase in resource envelope level.

#### Total pushable flow Total pushable flow Complexity Key Shortest distance to arepsilonDistance label Distance label Maximum Flow Algorithms Time Complexity O(NE logU) O(NEU) Successive shortest (O(N2E) Generic Preflow-gush | O(NPE) (<sub>N</sub>)O FIFO Preflow-push Capacity scaling \*Algorithm Labeling paths





Maximum Resource-Level
Increment Predecessor Set
Theorem 1: P<sub>max</sub> = set of events that is reachable
from or in the residual network of a f<sub>max</sub>
Theorem 2: P<sub>max</sub> is unique and has the minimal
number of events

#### Separation Schedule and Separation Time

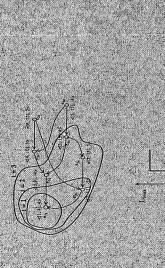
We know how to compute a P<sub>max</sub> but ...

events in C<sub>H</sub> and P<sub>max</sub> are schedule at or before t<sub>x</sub> and ... given a P<sub>max</sub> is there a temporally consistent schedule and a time  $t_{\chi}$  such that all all events in Pemax and OH are scheduled after t.?

Theorem 3: Yes!"



## Building a full envelope



## Maximum Resource Level and Resource Envelope

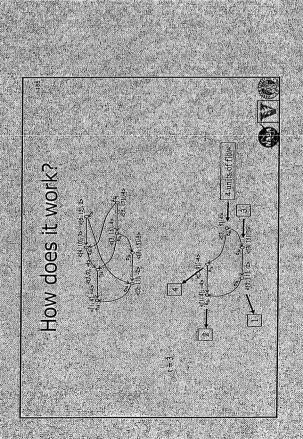
- Complete envelope profile [Muscettola, CP 2002]
  - Complexity: O(n O(maxflow(n, m, U))+nm)  $= L_{max}(t) = \Delta(G_0) + \Delta(P_{max}(R_t))$   $= P_{max}(R_t) \text{ and } C_t \text{ change only at et(e), and } H(e).$
- Can we do better?



## Staged Resource Envelope

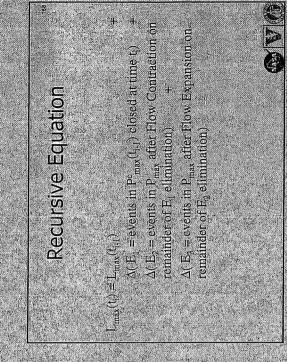
- Do not repeat flow operations on portion of the network that has already been used to compute envelope levels
- Deletion of flow due to elimination of consumers at time, out do not cause perturbation to incremental flow
- We can reuse much (all?) of the flow computation at previous stages, increasing performance

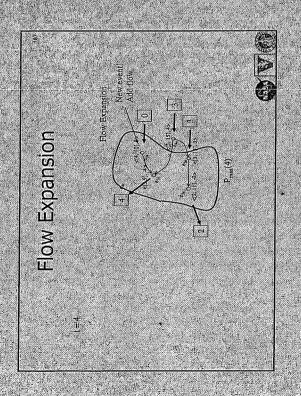




Flow Rightiction

Flow Contraction



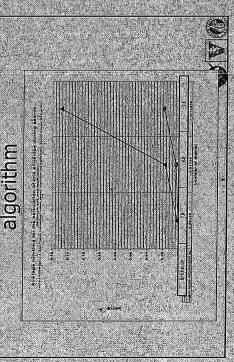


## Complexity Analysis

- Look at all known Maximum Flow algorithms
  - Identify complexity key
- Total pushable flow (Labeling methods)
- Shotrest distance to t. (Successive Shortest Paths):
   Distance label (Preflow-push methods)
- Show that complexity keys have same monotonic properties across multiple envelope stages that over a computation of maximum flow over entire network
  - Hence, complexity is O(Maxflow(n, mr U))



# Empirical speedup of staged



# Summarized excerpt from helpful comments of friendly ICAPS 2004 reviewers

"Sure, nice theory. But theory ain't much. Where are the empirical results, eh?"



## Envelope scheduling so far

- Policella et al. 2004]
- Non-backtrack, non-randomized commitment procedure = einet icInds a schedule at the first trial or it never will
  - Two kinds of contention profiles tested
    - Resource envelopes
- resource are profiles obtained by schedule executing all activities as early as possible.
  - Methods using earliest start profiles perform better on tested benchmark
- Open problem: is there other structural information in the envelopes that can be useful outside of contention identification?





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ve list only some initial sources for ideas and, where avaiable, survey papers that provide detail and additional references; these survey papers are in boldface and color.

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